

## Problem 1.37

[Difficulty: 1]

**1.37** An important equation in the theory of vibrations is

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = f(t)$$

where  $m$  (kg) is the mass and  $x$  (m) is the position at time  $t$  (s). For a dimensionally consistent equation, what are the dimensions of  $c$ ,  $k$ , and  $f$ ? What would be suitable units for  $c$ ,  $k$ , and  $f$  in the SI and BG systems?

**Given:** Equation for vibrations.

**Find:** Dimensions of  $c$ ,  $k$  and  $f$  for a dimensionally consistent equation. Also, suitable units in SI and BG systems.

**Solution:** Use the vibration equation to find the dimensions of each quantity

The first term of the equation is  $m \cdot \frac{d^2x}{dt^2}$

The dimensions of this are  $M \times \frac{L}{t^2}$

Each of the other terms must also have these dimensions.

Hence

$$c \cdot \frac{dx}{dt} = \frac{M \cdot L}{t^2} \quad \text{so} \quad c \times \frac{L}{t} = \frac{M \cdot L}{t^2} \quad \text{and} \quad c = \frac{M}{t}$$

$$k \cdot x = \frac{M \cdot L}{t^2} \quad \text{so} \quad k \times L = \frac{M \cdot L}{t^2} \quad \text{and} \quad k = \frac{M}{t^2}$$

$$f = \frac{M \cdot L}{t^2}$$

Suitable units for  $c$ ,  $k$ , and  $f$  are

$$c: \quad \frac{\text{kg}}{\text{s}} \quad \frac{\text{slug}}{\text{s}} \quad k: \quad \frac{\text{kg}}{\text{s}^2} \quad \frac{\text{slug}}{\text{s}^2} \quad f: \quad \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \quad \frac{\text{slug} \cdot \text{ft}}{\text{s}^2}$$

Note that  $c$  is a damping (viscous) friction term,  $k$  is a spring constant, and  $f$  is a forcing function. These are more typically expressed using  $F$  (force) rather than  $M$  (mass). From Newton's 2nd law:

$$F = M \cdot \frac{L}{t^2} \quad \text{or} \quad M = \frac{F \cdot t^2}{L}$$

Using this in the dimensions and units for  $c$ ,  $k$ , and  $f$  we find

$$c = \frac{F \cdot t^2}{L \cdot t} = \frac{F \cdot t}{L} \quad k = \frac{F \cdot t^2}{L \cdot t^2} = \frac{F}{L} \quad f = F$$

$$c: \quad \frac{\text{N} \cdot \text{s}}{\text{m}} \quad \frac{\text{lbf} \cdot \text{s}}{\text{ft}} \quad k: \quad \frac{\text{N}}{\text{m}} \quad \frac{\text{lbf}}{\text{ft}} \quad f: \quad \text{N} \quad \text{lbf}$$